MATHEMATICS
Adolescence and Young Adulthood ♦ Ages 14–18+

Entry 1:
Developing and Assessing Mathematical Thinking and Reasoning

Entry 2:
Instructional Analysis: Whole Class Mathematical Discourse

Entry 3:
Instructional Analysis: Small Group Mathematical Collaborations

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Entry 1: Developing and Assessing Mathematical Thinking and Reasoning

This entry is the student work entry. The main objective is to demonstrate your ability to design and implement a unit of mathematics building on students’ current knowledge and understanding to develop and enhance students’ abilities to think and reason mathematically. Teachers should choose an appropriate learning goal or target for their students and develop an instructional sequence designed to assess students’ prior knowledge and build on it to deepen students’ mathematical thinking and enrich their mathematical skills.

a. Knowledge of Students (KOS)

- Consider the following sources for your collection of data:
  - Building profile, school report card, state assessments, associate principal, building office personnel, district office, district or school website, other education professionals and IEPs.
  - Surveys of non-confidential issues: pre/post surveys, learning styles, interest inventories, personalities, etc.
  - Internet search on your city for community data

- Although each section below may affect what you teach and how you teach your specific students, which of the following sections are specific to your setting and necessary to demonstrate student impact? How do these affect your instruction? Remember: some of this information may be included in your contextual information.
  - Gender, age, grade in school
  - Community (e.g., urban/suburban/rural, levels of affluence, geographical location, cultural life experiences)
  - Academic ranges
  - Student interests
  - Special needs – any that apply
  - Family (e.g., economics, free/reduced lunch, parent education, family structure)
  - Ethnicity/cultural issues
  - Personality of the class
  - Relevant features (e.g., available resources, schedules, course sequence)
  - Instructional challenges: How do they approach assignments? Do they think and reason or just follow a pattern? Do they have the skills but need to develop the reasoning to truly problem solve?

b. Goals/Connections (G/C)

- What is the important mathematical concept and the learning goals/targets for the entire instructional sequence?
  - What do the students already know before you begin?
  - Is this a learning sequence where students are required to demonstrate more than just the development of key mathematical skills? Are they required to communicate their understanding of the key mathematical concept requiring them to implement said skills?
§ What modifications/interventions will you implement if students don’t grasp the concept the first time through the instructional sequence?
§ Use NCTM’s Principles and Standards for School Mathematics as a resource for identifying the important mathematical concept as well as identifying learning goals/targets designed to enhance students’ understanding. Other resources would be NCTM publications like Mathematics Teacher or Mathematics Teaching in the Middle School, state assessment outcomes and learning indicators; College Board’s SAT/ACT required concepts and/or skills; Advanced Placement course topics.
§ Consider several learning goals/targets for the entire instructional sequence that support and facilitate student understanding. These may extend beyond what you will feature in the entry. What will the student be doing/demonstrating/communicating?
§ Consider what instructional supports are needed to develop complex thinking and problem-solving strategies.

c. Instruction (INS)
  ❖ Does your instructional sequence consider:
    § Prior knowledge and skills
    § Instructional activities leading up to and between the featured activities
    § An incremental and logical progression of skills and processes.
    § Meeting the needs and/or learning styles of all students.
  ❖ When selecting the two featured activities consider
    § Are the activities central and critical to the big idea or mathematical concept?
    § Do these activities demonstrate mathematical reasoning or reveal how students were thinking about the idea before and then after the activities?
    § Are students demonstrating and applying mathematical skills and processes that they have developed in the instructional sequence as a foundation for their overall conceptual understanding of the mathematical idea? Are they communicating their critical thinking process and/or conceptual understanding?
    § Does each activity connect to each other? Does each activity meet the needs and/or learning styles of all students?
  ❖ What are the links between your assessment and your featured activities?
    § How is the student's progress revealed?
    § Did you design the pre- and post-assessments before you begin planning the instructional sequence?
    § How did you meet the needs and learning styles of all your students?
    § Explain how you used feedback to support student learning.
  ❖ What are the challenges and/or stumbling blocks in learning the important mathematical concept and/or in completing the instructional sequence of activities that typically occur?
    § Can you tie your knowledge of teaching mathematics to specific knowledge about this particular group of students?
Mathematics: Adolescence and Young Adulthood

- Did you modify any activities in response to student needs?
- Are there common mistakes and errors students make in learning this mathematical idea? Were you able to anticipate/prevent? What did you have to specifically re-teach to correct, etc.?

d. Analysis (ANA)
   - The information should be presented in separate sections for each student.
   - Provide contextual information to bring the student to life. Describe the unique characteristics of this student.
   - What is each student's strengths and weaknesses, gaps in prior knowledge and misconceptions from which he/she operates? How does this impact their ability to grasp the big mathematical concept.
   - How did each student progress through the featured activities? How did you assess their level of understanding of the concept, their progress, their initial misconceptions, etc.?
   - What specific evidence in the student work can you cite exemplifying their ability to critically think as they progressed through the learning sequence?

e. Assessment (ASMT)
   - Throughout this process, keep in mind that the National Board assessor is looking for evidence that you use a variety of assessment strategies in order to improve your work with students. While this is not a specific question to be addressed, throughout your commentary you should provide evidence in your instructional sequencing that you employ various strategies and use the results to adjust/modify/enrich instruction.

f. Feedback Next Steps (FB)
   - The information should be presented in separate sections for each student.
   - What feedback did you give to students and how did you formulate your next steps for math instruction?
      - Provide specific evidence of the nature and content of the feedback, including written and verbal comments.
      - How did the feedback improve student ability and help the student to achieve the instructional goals?
      - If you provide verbal feedback, you will need to document this as well, possibly by writing it on the paper copies of the activities and then tying this into your commentary.

g. Content Knowledge (CK)
   - Throughout your commentary, is your discussion, vocabulary, notation, etc., mathematically accurate and course appropriate?
   - In your instructional sequencing and decision-making, as well as your student analysis, how did you use sound pedagogy for your level/group of students?
h. Reflection (R)

- Consider next steps for instruction.
  - How did you identify and explain what the next steps should be as indicated by the evidence of student performance?
  - If the students were successful in attaining the goal, what will be the next logical step in increasing their understanding of the important mathematical concept?
  - If the students were not successful in attaining the goal, what will be done to remedy the situation?
  - Based on the evidence, are the two students featured in the entry typical within the whole group or not?
- Is there evidence the students’ responses that suggests you should do something differently next time?
- Analyze your teaching practice for the strengths and weaknesses revealed through this instructional sequence.
- What activities would you repeat using your analysis of the student work? What would make the lesson even better the next time?
- How would you modify your teaching as a result of this activity?
Entry 2: Instructional Analysis: Whole Class Mathematical Discourse

This entry is a whole group video entry that demonstrates how you facilitate a classroom discussion using targeted questioning to develop student understanding about an important mathematical idea. You will demonstrate your ability to engage students in mathematical discourse as the whole class investigates, explores, or discovers important mathematical concepts, procedures, or reasoning processes that promote student learning.

a. Knowledge of Students (KOS)

- Consider the following sources for your collection of data:
  - Building profile, school report card, state assessments, associate principal, building office personnel, district office, district or school website, other education professionals and IEPs.
  - Surveys of non-confidential issues: pre/post surveys, learning styles, interest inventories, personalities, etc.
  - Internet search on your city for community data

- Although each section below may affect what you teach and how you teach your specific students, which of the following sections are specific to your setting and necessary to demonstrate student impact? How do these affect your instruction? Remember: some of this information may be included in your contextual information.
  - Gender, age, grade in school
  - Community (e.g., urban/suburban/rural, levels of affluence, geographical location, cultural life experiences)
  - Academic ranges
  - Student interests
  - Special needs – any that apply
  - Family (e.g., economics, free/reduced lunch, parent education, family structure)
  - Ethnicity/cultural issues
  - Personality of the class
  - Relevant features (e.g., available resources, schedules, course sequence)
  - Instructional challenges: How do they approach assignments? Do they think and reason or just follow a pattern? Do they have the skills but need to develop the reasoning to truly problem solve?

b. Goals/Connections (G/C)

- What is the important mathematical concept and the learning goals/targets for the entire instructional sequence?
  - Is this a learning sequence where students are required to demonstrate more than just the development of key mathematical skills? Are they required to communicate their understanding of the key mathematical concept requiring them to implement said skills?
- Use NCTM’s Principles and Standards for School Mathematics as a resource for identifying the important mathematical concept as well as identifying learning goals/targets designed to enhance students’ understanding. Other resources could include National/state/district curriculum standards, course outline, state assessment guidelines, College Board Standards.
- Consider several learning goals/targets for the entire instructional sequence that support and facilitate student understanding. These may extend beyond what you will feature in the video. What will the students be doing/demonstrating/communicating?
- Consider what instructional supports are needed to develop complex thinking and problem-solving strategies.
- What is the link between preceding topics in mathematics to the lesson being observed in the video? Describe how you made that link.
- Identify CLEARLY the mathematical “big idea” for the entry.
- When planning, be able to answer the following questions: “What do I do if they already know the material?” “What do I do if they don’t grasp the concept the first time through the instructional process?”
- What is each goal designed to do?
- How do you link each goal to the unit goal (big idea) and to the needs of the student?
- Why does a whole class discourse format best support your goals?

c. Content Knowledge (CK)
- How does your content knowledge and knowledge of mathematics pedagogy affect
  - Sequencing of the lesson within the curriculum
  - Goals of the unit and goals of the lesson
  - How the whole class discussion is facilitated
- Why was this lesson taught through whole group discourse and how are the needs of the students met through this instruction?
- How did your content knowledge allow you to correct and redirect misconceptions during the discussion

d. Instruction (INS)
- Consider the following:
  - The sequencing of the lesson within the curriculum unit. What considerations must be addressed to make this determination?
  - The advantage of the whole group to enhance student learning of this particular topic.
  - Key interactions or concepts that would indicate to you that the students are using mathematical reasoning and communicating mathematically.
e. Learning Environment (LE)
   - Is the learning environment equitable, accessible and fair?
     - What evidence shows that students are willing/able to communicate different viewpoints without repercussion?
     - Are different learning styles (if applicable) accepted and integrated within the class?
   - Is the learning environment productive and conducive to mathematical reasoning and discourse (including questioning and probing)? Consider
     - Students sharing viewpoints or discoveries.
     - Students participate within the group and listen to each other.
     - Students can negotiate mathematical ideas, ask each other for clarification, and/or be willing to teach each other within the group.
     - When videotaping, make sure you can be seen on the video at some point in time in your selection. Primarily, the whole class needs to be visible at all times.

f. Engagement (ENG)
   - Students should have a strategy for building ideas based on the group’s discussion. They should be trained to assist each other and articulate their discovery. Does the discourse answer the following types of questions:
     - Mathematical reflecting: “What did you do first? Why? What were you thinking when you recorded this? Which clue did you think was the most (least) helpful? Why? What made this investigation easy (difficult) for you? What could you do next? What hint would you give to a friend that was stuck?”
     - Mathematical connections: “Does this problem remind you of another problem that you’ve already solved? Is there another way to solve this problem? Can you create a problem that could also be solved this way? Can you represent this information in a different way?”
     - Mathematical predictions: “What do you think will happen next? Why? Do you think this pattern will continue? Why? Would this still be true if you began with an odd number (or other counterexample)? Can you state a general rule that you have discovered? What will never happen when you do this?”
   - Do students demonstrate inquiry from each other and do they demonstrate their willingness to help each other? Do the students take responsibility for their own learning (e.g., “I don’t understand” followed by “Here, let me show you.”).

g. Assessment (ASMT)
   - Considerations:
     - Interacts with the whole class
     - Facilitates discussion so that it is truly student-driven
     - Determines understanding without intervening in the group’s discourse
     - Gets students back on track if they are not understanding
     - Extends the lesson if the students are understanding
h. Analysis (ANA)

- It can be helpful to create a transcript or storyboard of the video, marking time in the margins for reference. Knowing where you had discussions with student’s using questioning techniques to help their mathematical reasoning is critical. When specifying students in the video, utilize their name and color of attire. Once the color of attire is specified, you can then just utilize their name. Select specific examples and quotes from the video that demonstrate specific questioning/prompting techniques you used. Be sure to also identify non-verbal cues. It is critical to show active verbal exchange and nonverbal signs of engagement. Facial expressions and body language are critical indicators and vary by individual.
- Why is the whole group format essential to learning the goals of this unit? Are there specific things that would not have occurred if you had used another method?
- How were the learning goals achieved?
- Were there specific video segments where your ability to perform informal assessments of learning impacted the whole class discussion? Show how your content knowledge allowed you to correct and redirect misconceptions during these discussions. Were you able to predict which students would struggle at a particular point and provide interventions at that point? Where were those and what did you know to do because you know those students so well?
- Use student words and the flow of the conversation to demonstrate students had ownership of the conversation…that you were aware of unanticipated opportunities this ownership provided and what that revealed about not only the students’ understanding of the mathematical idea but their ability to negotiate and connect ideas. Analyze the student-to-student as well as student-to-teacher discourse.
- Did you make any modifications in response to student needs? How did you know to make those modifications and what was the impact of doing those on the whole group discourse?
- Cite and explain specific video segments that prove students achieved the learning and also were able to engage in mathematical discourse with each other and you (the teacher).

i. Feedback (FB)

- What feedback did you provide? Remember, feedback is not always verbal.
- Make note of the feedback given from other students.

j. Reflection (R)

- Identify moments where student understanding and ownership of the discourse were at a peak. Explain why your expertise in either content knowledge or pedagogy influenced the direction the lesson took. Justify the choices you made while the lesson was going on that shows your expertise.
Reflect on your interaction with students. What strengths and weaknesses does it show about your teaching practice? Which segments of student words/actions were successful and explain why you consider these successful? Cite video segments with student words/action that could be improved in future lessons and explain why.

What would you do differently and why? Also, look at what went well and why it worked.
Entry 3: Instructional Analysis: Small Group Mathematical Collaborations

a. Knowledge of Students (KOS)

- Consider the following sources for your collection of data:
  - Building profile, school report card, state assessments, associate principal, building office personnel, district office, district or school website, other education professionals and IEPs.
  - Surveys of non-confidential issues: pre/post surveys, learning styles, interest inventories, personalities, etc.
  - Internet search on your city for community data

- Although each section below may affect what you teach and how you teach your specific students, which of the following sections are specific to your setting and necessary to demonstrate student impact? How do these affect your instruction? Remember: some of this information may be included in your contextual information.
  - Gender, age, grade in school
  - Community (e.g., urban/suburban/rural, levels of affluence, geographical location, cultural life experiences)
  - Academic ranges
  - Student interests
  - Special needs – any that apply
  - Family (e.g., economics, free/reduced lunch, parent education, family structure)
  - Ethnicity/cultural issues
  - Personality of the class
  - Relevant features (e.g., available resources, schedules, course sequence)
  - Instructional challenges: How do they approach assignments? Do they think and reason or just follow a pattern? Do they have the skills but need to develop the reasoning to truly problem solve?

b. Goals/Connections (G/C)

- What is the important mathematical concept and the learning goals/targets for the entire instructional sequence?
  - Is this a learning sequence where students are required to demonstrate more than just the development of key mathematical skills? Are they required to communicate their understanding of the key mathematical concept requiring them to implement said skills?
  - Use NCTM’s Principles and Standards for School Mathematics as a resource for identifying the important mathematical concept as well as identifying learning goals/targets designed to enhance students’ understanding. Other resources could include National/state/district curriculum standards, course outline, state assessment guidelines, College Board Standards.
Consider several learning goals/targets for the entire instructional sequence that support and facilitate student understanding. These may extend beyond what you will feature in the video. What will the students be doing/demonstrating/communicating?

Consider what instructional supports are needed to develop complex thinking and problem-solving strategies.

What is the link between preceding topics in mathematics to the lesson being observed in the video? Describe how you made that link.

Identify CLEARLY the mathematical “big idea” for the entry.

When planning, be able to answer the following questions: “What do I do if they already know the material?” “What do I do if they don’t grasp the concept the first time through the instructional process?”

What is each goal designed to do?

How do you link each goal to the unit goal (big idea) and to the needs of the student.

Why does a whole class discourse format best support your goals?

c. Instructional (INS)

Consider the following:

The sequencing of the lesson within the curriculum unit. What considerations must be addressed to make this determination?

The advantage of the small group to enhance student learning of this particular topic.

Key interactions or concepts that would indicate to you that the students are using mathematical reasoning and communicating mathematically.

Describe any techniques used to create the small groups (i.e., color theories, birth order, self-assessments on leadership roles). Be specific with examples showing how these methods were demonstrated in the small group.

You may also describe any techniques used previously that were not successful if you deviate from what the theories suggest you should do.

Groups do not always work the same. Use your knowledge of students to identify and match students with similar small group styles for this lesson.

Why was the use of small groups essential to this particular exploration? How does this relate to students’ mathematical understanding of the concept?

d. Content Knowledge (CK)

How does your content knowledge and knowledge of mathematics pedagogy affect

Sequencing of the lesson within the curriculum

Goals of the unit and goals of the lesson

How the whole class discussion is facilitated

Why was this lesson taught through whole group discourse and how are the needs of the students met through this instruction?
How did your content knowledge allow you to correct and redirect misconceptions during the discussion?

e. Learning Environment (LE)

- Is the learning environment equitable, accessible and fair?
  - What evidence shows that students are willing/able to communicate different viewpoints without repercussion?
  - Are different learning styles (if applicable) accepted and integrated within the class?

- Is the learning environment productive and conducive to mathematical reasoning and discourse (including questioning and probing)? Consider
  - Students sharing viewpoints or discoveries.
  - Students participate within the group and listen to each other.
  - Students can negotiate mathematical ideas, ask each other for clarification, and/or be willing to teach each other within the group.

- Is the learning environment safe and inclusive?
  - Establish the evidence of why the groups are set the way they are.
  - Demonstrate evidence that different learning styles (if applicable) are accepted and integrated within the small group.

f. Engagement (ENG)

- Students should have a strategy for building ideas based on the group’s discussion. They should be trained to assist each other and articulate their discovery. Does the discourse answer the following types of questions:
  - Mathematical reflecting: “What did you do first? Why? What were you thinking when you recorded this? Which clue did you think was the most (least) helpful? Why? What made this investigation easy (difficult) for you? What could you do next? What hint would you give to a friend that was stuck?”
  - Mathematical connections: “Does this problem remind you of another problem that you’ve already solved? Is there another way to solve this problem? Can you create a problem that could also be solved this way? Can you represent this information in a different way?”
  - Mathematical predictions: “What do you think will happen next? Why? Do you think this pattern will continue? Why? Would this still be true if you began with an odd number (or other counterexample)? Can you state a general rule that you have discovered? What will never happen when you do this?”

- Do students demonstrate inquiry from each other and do they demonstrate their willingness to help each other? Do the students take responsibility for their own learning (e.g., “I don’t understand” followed by “Here, let me show you.”).
g. Assessment (ASMT)

- Considerations:
  - Interacts with the small group
  - Facilitates discussion so that it is truly student-driven
  - Determines understanding without intervening in the groups’ discourse
  - Gets students back on track if they are not understanding
  - Extends the lesson if the students are understanding

h. Technology/Manipulatives (T/M)

- Discuss the significance of teacher’s use of technology/manipulatives in promoting students’ mathematical understanding. The teacher must show the concept under investigation could not have been taught as effectively without the manipulatives or technology incorporated. The teacher must demonstrate how the learning is enhanced or even made possible, how it allows the learner to make conceptual connections, or how it allows students to teach each other through technology or manipulatives.

i. Analysis (ANA)

- It can be helpful to create a transcript or storyboard of the video, marking time in the margins for reference. Knowing where you had discussions with students using questioning techniques to help their mathematical reasoning is critical. When specifying students in the video, utilize their name and color of attire. Once the color of attire is specified, you can then just utilize their name. Select specific examples and quotes from the video that demonstrate specific questioning/prompting techniques you used. Be sure to also identify non-verbal cues. It is critical to show active verbal exchange and nonverbal signs of engagement. Facial expressions and body language are critical indicators and vary by individual.

- Why is the small group format essential to learning the goals of this unit? Are there specific things that would not have occurred if you had used another method?

- How were the learning goals achieved?

- Were there specific video segments where your ability to perform informal assessments of learning impacted the small class discussion? Show how your content knowledge allowed you to correct and redirect misconceptions during these discussions. Were you able to predict which students would struggle at a particular point and provide interventions at that point? Where were those and what did you know to do because you know those students so well?

- Use student words and the flow of the conversation to demonstrate students had ownership of the conversation…that you were aware of unanticipated opportunities this ownership provided and what that revealed about not only the students’ understanding of the mathematical idea but their ability to negotiate and connect ideas. Analyze the student-to-student as well as student-to-teacher discourse.
Did you make any modifications in response to student needs? How did you know to make those modifications and what was the impact of doing those on the whole group discourse?

Cite and explain specific video segments that prove students achieved the learning and also were able to engage in mathematical discourse with each other and you (the teacher).

Cite and explain the critical role the manipulative/technology played in accomplishing your learning goal. Use specific student words, actions or discoveries that wouldn’t have been possible without the manipulative or technology.

j. Reflection (R)

Identify moments where student understanding and ownership of the discourse were at a peak. Explain why your expertise in either content knowledge or pedagogy influenced the direction the lesson took. Justify the choices you made while the lesson was going on that shows your expertise.

Reflect on your interaction with students. What strengths and weaknesses does it show about your teaching practice? Which segments of student words/actions were successful and explain why you consider these successful? Cite video segments with student words/action that could be improved in future lessons and explain why.

What would you do differently and why? Also, look at what went well and why it worked.