Mathematics, Nature, and Rep-tiles

Mathematics and nature go hand-in-hand. In other words, mathematics is used to help describe natural phenomena, whereas natural structures in their ideal forms provide physical models of mathematical concepts. One such example is that of tilings, many times referred to as tessellations. A tiling is a covering of the plane with copies of geometric figures called tiles in such a way that there are no gaps or overlaps between tiles. Copies of a regular hexagon will tile the plane. Nature provides us with many examples of a hexagonal tiling. Probably the most famous of these examples is the cross-section of a honeycomb. Any quadrilateral will tile the plane. The cross-section of the onion root tip provides an example of a tiling by rectangles (See Figure 1).

![Figure 1](image)

As can be seen in Figure 2, the scales of a snake skin ideally form parallelograms, which can be used to tile a plane. Notice that four parallelograms fit together to form a larger similar parallelogram. Four of these larger parallelograms fit together to form yet an even larger parallelogram similar to both the original and the intermediate parallelograms. If we continue this replication, building even larger figures, we can tile any plane. Golomb (1964) refers to this type of tile as a rep-tile.
A rep-tile is a geometric figure such that copies of the figure fit together to form a larger similar figure. An interesting fact is that any triangle and any parallelogram is a rep-4 tile. That is, four copies of a triangle or parallelogram fit together to form a larger similar triangle or parallelogram (See Figure 3).

Another interesting fact that can be seen in Figure 4 is that these rep-4 tiles are also rep-9 tiles; that is, nine congruent tiles fit together to form a larger similar figure.
In this discussion, only triangles and parallelograms are considered. However, for those wishing additional information about rep-tiles, see Golomb (1964), Gardner (1991), and watch for an upcoming article by the authors to appear in the *Arithmetic Teacher* (Fosnaugh and Harrell, In press).

In the mathematics classroom, teachers can provide many examples of tilings in nature. One way is by using physical examples such as a jar of honey with a honeycomb, wasps' nests, snake skins, or microscope slides upon which biological specimens have been mounted. Although mathematics teachers might not have these available in their classrooms, they can find them through scavenger hunts, in grocery stores, and from local science teachers. Teachers also should not forget how resourceful their students can be in collecting specimens. Many school districts have a resource center with microscopes that can be checked out. If not, check again with your local science teacher. Alternatively, use pictures from scientific and nature magazines, science books, and catalogs from science vendors.

Middle school students have found the following activity, taken from a learning center, to be enjoyable yet challenging. Before using this activity, we have found it helpful to review the notions of similar figures and tilings (tessellations). Depending upon the available resources, this activity can be modified for students in lower grades or can be adapted for use in a high school geometry class. In using this activity, we have noticed that by approaching concepts in mathematics through the realm of nature (science), we not only integrate teaching mathematics and science, but we also provide the students with applications of mathematics to the outside world. In turn, the students develop a better appreciation of the mathematical concepts being investigated.

**TILINGS IN NATURE**

Part 1

In this center, you will find snake skins, a jar of honey, wasps' nests, microscope slides, and several photographs of different objects found in nature. For each of the objects in the center, determine if a part of the object or photograph can be duplicated and used to tile the plane. For each geometric figure that will tile the plane,
draw a diagram showing how it tiles the plane. For the others, explain why they cannot be used to tile the plane. Remember that in nature, things are not perfect. Think of these shapes, however, as being congruent when they seem to have the same general shape.

Part 2

Notice that the snake skins appear to be made up of parallelograms. Four copies of the same parallelogram fit together to form a larger similar parallelogram. Four of these larger similar figures fit together to form an even larger similar parallelogram. If we continue this process indefinitely, the plane can be tiled. See the picture below.

Because it takes four congruent tiles to create a larger similar figure, the parallelogram is called a rep-4 tile.

Draw a triangle and determine if it is a rep-4 tile. Draw another triangle which is different from the previous one. Determine if this triangle is a rep-4 tile. Do you think that every triangle is a rep-4 tile? Explain.

Mathematicians have shown that these rep-4 tiles are also rep-9 tiles. Using a triangle, show that it is a rep-9 tile. Repeat this with a parallelogram.

Resources


Literature Cited


Linda S. Fosnaugh and Marvin E. Harrell
Division of Mathematics
Emporia State University