The Geometer's Sketchpad and Classroom Activities
by
C. Bryan Dawson and Connie Schrock

In the summer of 1995, the authors taught a course entitled "Learning Geometry using the Geometer's Sketchpad." The course was designed to give classroom teachers hands-on experience with the Sketchpad, as well as with the activities in the text Exploring Geometry with the Geometer's Sketchpad. [1] The Geometer’s Sketchpad is a dynamic tool for exploring geometry. Students and teachers can use it to explore and manipulate Euclidean, coordinate, transformational, analytic and fractal geometry. The Geometer's Sketchpad truly encourages learning from an exploratory perspective. Students can first visualize and analyze a problem before they are asked to create proofs.

The aforementioned text begins with four guided tours to introduce students to the capabilities of the software package. These tours are appropriate for students, and the vast majority of the activities can be completed with no further instruction on software usage. The remainder of the book consists of blackline activity masters which cover a wide range of high school geometry concepts. The activities are divided by topics and by style, which include investigations, explorations, constructions, problems, art and a puzzle. In addition, Key Curriculum Press has published a variety of other resources [2-6] which focus on the use of the Geometer’s Sketchpad in other mathematics courses such as trigonometry.

The general consensus of the classroom teachers involved in the course is that activities in the text were good. One must remember to carefully select which activities to use and under what circumstances to use them, so that this method of student learning is integrated into the curriculum without slowing down the progress of the course.

When students are asked to use the software, they often want to do self-exploration rather than follow step-by-step directions provided by the teacher. Teachers must be aware of the types of learners in their classes and how they will approach these activities. Even with the many resources available to teachers, they will often find the need to modify or write activities of their own. However, by using the supplementary resources and knowing how to use the Geometer’s Sketchpad teachers can meet curriculum goals in an exciting way.

One of the difficulties that arises when working with teachers on activities using the Geometer’s Sketchpad is the activities in the workbook are designed for students who have very little previous knowledge of the geometry involved. Teachers who use the activities in a class such as this have taught most or all of the geometric concepts involved. To help overcome this roadblock, we designed several investigations that are similar in format to the ones found in Exploring Geometry with the Geometer’s Sketchpad, but they contain geometric concepts not necessarily known by most classroom teachers. Six are given here. Try them for yourself. Many of the activities are appropriate for your students.

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Investigation 1: Fermat’s Point

In this investigation you’ll discover Fermat’s point. This is the same Fermat of Fermat’s Last Theorem.

Sketch
Step 1: Construct a triangle $ABC$.

Step 2: Construct an equilateral triangle outward from triangle $ABC$ using $AB$ as one of its sides.

Step 3: Construct the circumcircle of the equilateral triangle you constructed in Step 2.

Step 4: Repeat steps 2 and 3 for sides $BC$ and $AC$.

Investigate
What do you notice as you drag the vertices of triangle $ABC$? Does it matter if the triangle is right, obtuse, or acute? If you think you have found Fermat’s point, under what conditions is it inside triangle $ABC$? outside triangle $ABC$? on triangle $ABC$?

Conjecture: Write your conjecture(s) below.

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Explore More
Plot a point somewhere on your diagram and measure its distance from each of the vertices of triangle $ABC$. Calculate the sum of these distances. Do you notice anything as you move this point around?

Designed by Dr. C. Bryan Dawson, Emporia State University.
Investigation 2: An Open Problem

Sketch
Step 1: Construct a triangle $ABC$.

Step 2: Place a point, $P$, inside the triangle.

Step 3: Construct lines (or rays) $AP$, $BP$, and $CP$ and their intersections ($D$, $E$, and $F$, respectively) with the sides of the triangle.

Step 4: Construct triangle $DEF$.

Investigate
Can you make triangle $DEF$ equilateral by dragging point $P$? Do some measuring to verify your results. Drag points $A$, $B$, and $C$, and try it again.

Conjecture: Write your conjecture(s) below.

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Explore More
The problem of finding all points $P$ such that triangle $DEF$ is equilateral (with proof) appeared as *American Mathematical Monthly* problem 10358 in the January, 1994 issue. At that time, no solution was known! A solution appeared in the June-July 1997 American Mathematical Monthly (Vol. 104 No.6, pp. 567-570).
Investigation 3: Ceva’s Theorem

In this investigation you’ll discover Ceva’s Theorem. Ceva was an Italian mathematician; he discovered this theorem in approximately 1678.

Sketch
Step 1: Construct triangle $ABC$.

Step 2: Place a point, $D$, inside the triangle.

Step 3: Construct lines (or rays) $AD$, $BD$ and $CD$ and their points of intersection with the sides of the triangle.

Step 4: Hide (and reconstruct?) necessary objects.

Step 5: Measure and find the ratios $AG$, $GB$, $BE$, $EC$, $CF$, and $FA$, and find the ratios $\frac{AG}{GB}$, $\frac{BE}{EC}$, and $\frac{CF}{FA}$.

Investigate
Drag point $D$. If you produced your sketch correctly, the three segments $AE$, $BF$, and $CG$ should still pass through point $D$. What happens to the three ratios you calculated? Perform some calculations on the three ratios and see if you can build an expression that remains constant as point $D$ is moved.

Conjecture: Write your conjecture(s) below.

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Explore More
Change your sketch by extending the sides of the triangles to lines and allowing points $E$, $F$, and/or $G$ to stray outside the triangle when $D$ is moved outside the triangle. Does your conjecture remain true?
Investigation 4: The Steiner Line

In this investigation you’ll discover the Steiner Line. Jacob Steiner (1796-1863) discovered it in about 1860.

Sketch
Step 1: Draw four lines in such a way that four triangles are formed (in the figure at the right, the triangles are $ACE$, $BCD$, $DEF$, and $ABF$).

Step 2: Construct the orthocenters (intersection of the altitudes) of the four triangles.

Investigate
What do you notice about the four orthocenters? Try moving the four original lines to see if the relationship still holds. If you have found the relationship, try adding something to your sketch as a check. Were you right?

Conjecture: Write your conjecture(s) below.

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Present Your Results
If you haven’t done so already, clean up your sketch, put in appropriate labels, and add comments.

Explore More
Are the incenters, circumcenters, incircles, circumcircles, or centroids of the four triangles related in some way? Can you find Clifford’s point, discovered in 1860?

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Investigation 5: Using the Trace

Sketch
Step 1: Construct a line segment with endpoints A and B which contains a movable point C. Hide the original segment and construct new line segments AC and CB.

Step 2: Construct two points that are not on the given line segments. Use these points as the centers of two circles, one having a radius of AC and the other the same as CB. Make sure that the conditions are such that the circles intersect and construct the two points of intersection. Hide the circles.

Step 3: Select the two points of intersection and choose trace points.

Step 4: Before you investigate further what shape do you expect from the locus as you move C along AB?

Investigate
Move C back and forth along the line segment and look at the locus. Why is this happening? Change the length of AB, change the distances for D and E. What happens?

Explore More
Can you change what was done so that the locus will be a hyperbola? What are the steps you would change? Create a script for the hyperbola.

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Investigation 6: Action Button and Loci

Sketch
Construct a line and a point, A, not on the line. Hide the points which created your line. On the line create two points which may move freely on the line and label them D and E. Construct segment AD and its midpoint, F.

Investigate
1. Remembering the definition of parabola, based on a focus and directrix, continue the construction so that a point on the parabola would be created. Hide the extraneous lines.

2. Highlight the point created and use trace point. Move D. Is the parabola created? Move the directrix and focus. What changes occur in the parabola?

3. Move D back to the left side of the diagram and make sure E is to the right. First highlight D and then E. Select the action button from the Edit menu and select movement. Select any movement speed when the choice is given. I recommend slow. Double click on the button to start the motion. Reset by moving the points back to opposite sides of the screen.

Explore More
You may change the text on the button by using the labeling tool by double clicking and then renaming it. Also experiment with the labeling tool by creating a text window and putting in some instructions for this activity. Modify the instructions on the screen so that a student would know what to move so that the student might observe the change in the parabola when other changes are made. Can you use animation to create other conics?
References


