Scalar and Vector Products

Scalar (dot) Product

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \qquad \hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$$

The result of a dot product is a scalar quantity – magnitude only, no direction.

Vector (cross) Product

$$\vec{A}\times\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta \qquad \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

Determinant method for cross products

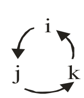
$$\vec{C}_x = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}\hat{i} = \left(A_y B_z - A_z B_y\right)\hat{i}$$

$$\vec{C}_y = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix}\hat{j} = -\left(A_x B_z - A_z B_x\right)\hat{j}$$

$$\vec{C}_z = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}\hat{k} = \left(A_x B_y - A_y B_x\right)\hat{k}$$

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \left(A_y B_z - A_z B_y\right)\hat{i} - \left(A_x B_z - A_z B_x\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

Cyclic method for cross products



$$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{i} = \hat{j} \qquad \hat{j}\times\hat{i} = -\hat{k} \qquad \hat{k}\times\hat{j} = -\hat{i} \qquad \hat{i}\times\hat{k} = -\hat{j}$$

$$\begin{aligned}\vec{A}\times\vec{B} &= \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) \\ &= A_x B_y\hat{k} + A_x B_z\left(-\hat{j}\right) + A_y B_x\left(-\hat{k}\right) + A_y B_z\hat{i} + A_z B_x\hat{j} + A_z B_y\left(-\hat{i}\right) \\ &= \left(A_y B_z - A_z B_y\right)\hat{i} + \left(A_z B_x - A_x B_z\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}\end{aligned}$$

The result of any cross product is a vector whose direction is perpendicular to both A and B.