PH 393 Physics II **Exam 3** **Fall 2011**

Name: Solutions **12/6/11**

Present neat and orderly answers for each question.
Clearly indicate your method of solution for each problem, including equations used.
Include appropriate units.
Show all work.

$\mu_0 = 1.26 \times 10^{-6}$ Tm / A

Multiple Choice (2 pts each)

1. A circuit contains a series combination of a resistor, a capacitor and an ac voltage source. The magnitude of the phase shift of the current is:

   $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$ at $t = 0$

   a. Zero;
   b. Less than $\pi/2$, but non-zero;
   c. Equal to $\pi/2$;
   d. Greater than $\pi/2$;

   Ans. B

2. A platform is placed on top of a solenoid. The plane of the platform is perpendicular to the long axis of the solenoid. An iron support rod, much longer than the length of the solenoid, is placed through a hole in the platform and passes through the center of the solenoid. An aluminum ring, with a diameter larger than that of the iron rod, is placed on the platform such that the iron rod passes through the center of the ring. If an ac voltage is connected across the solenoid, what happens when it is turned on?

   a. Nothing;
   b. The ring remains on the platform and heats up;
   c. The ring launches into the air and falls back to the platform;
   d. The ring launches into the air and falls back and levitates;

   Ans. D

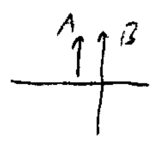
3. Inductors can be combined into an equivalent inductance similar to resistors and capacitors. Which combination of three identical inductors would give the largest inductance?

   Series: $L = L_1 + L_2 + L_3 + \cdots$
   Parallel: $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$

   a. Series;
   b. Series and parallel combination;
   c. Parallel;
   d. All combinations above will have the same inductance.

   Ans. A

4. A circular loop of wire resides in the plane of this page that is perpendicular to a uniform magnetic field. If the field points into the page and the angle between the magnetic field and the area vector increases in time from 0° to 90°, which of the following is true?

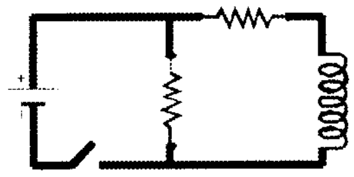
   $\theta = 0°$

   a. Φ_B increases and the induced current is clockwise;
   b. Φ_B increases and the induced current is counterclockwise;
   c. Φ_B decreases and the induced current is clockwise;
   d. Φ_B decreases and the induced current is counterclockwise.

   $0° < \theta < 90°$
   A decreases in time
   $\therefore \Phi_B$ decreases
   I is CCW relative to page

   Ans. D

5. The following circuit shows a circuit with two identical resistors and an inductor. A long time after closing the switch, how is the current through the central resistor related to the current through the other resistor?



$t \to \infty$

$\Delta V_L \to 0$

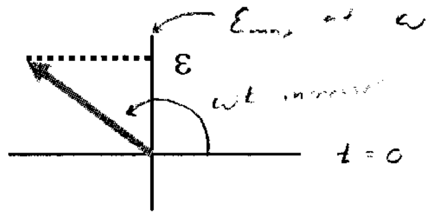
$I_1 = I_2$

a. the same as;
b. more than;
c. less than;
d. not enough information.

Ans. A

6. The magnitude of the instantaneous value of the emf represented by this phasor is

$\mathcal{E}_{max}$ at $\omega t = \frac{\pi}{2}$



a. Increasing;
b. Decreasing;
c. Constant;
d. Cannot answer without knowing time.

Ans. B

7. An AC current with maximum amplitude I_{max} = 4A supplies the same average power to a resistor as a DC current. The value of the DC current is:

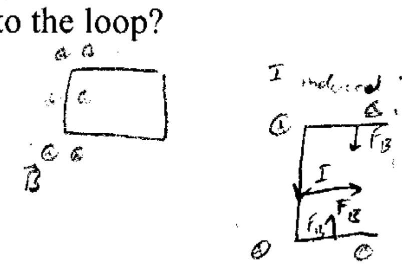
$I_{DC} = I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{4}{\sqrt{2}}$ A

a. 4 A;
b. $\frac{4}{\sqrt{2}}$ A;
c. 8 A;
d. $\frac{16}{\sqrt{2}}$ A.

Ans. B

8. A rectangular conducting loop has its left half in a magnetic field directed into the page. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?

$I_{induced} \to$ CCW



a. The loop is pushed upward;
b. The loop is pushed downward;
c. The loop is pushed left;
d. The loop is pushed right.

Ans. D

9. A transformer has 300 turns in the primary coil and 700 turns in the secondary coil. A voltage of 120 V AC is applied to the primary coil. A 50 Ω resistor is connected to the secondary coil, how much current is delivered to the primary coil?

$\frac{V_1}{N_1} = \frac{V_2}{N_2}$ $\quad V_2 = \frac{N_2}{N_1} V_1 = 280$ V

$I_2 = \frac{V_2}{R_2} = \frac{280 V}{50 \Omega} = 5.6$ A

$P_1 = P_2 \Rightarrow I_1 V_1 = I_2 V_2 \Rightarrow I_1 = \frac{I_2 V_2}{V_1} = 13.1$ A

a. 0 A;
b. 1.0 A;
c. 5.6 A;
d. 13.1 A.

Ans. D

10. A series RLC Circuit has V_C = 5.0 V, V_R = 7.0 V and V_L = 9.0V. How does the frequency of the circuit compare to the resonant frequency of the circuit?

$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\omega = \omega_0$ when $V_L = V_C$

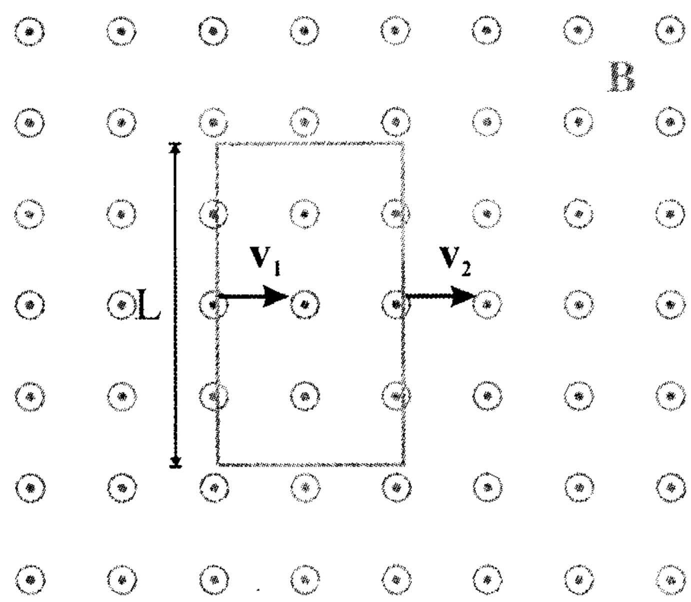
but $V_L > V_C$, so $X_L > X_C$

$\therefore \omega > \omega_0$

a. Higher;
b. Lower;
c. Equal to;
d. No comparison is possible.

Ans. A

Problem 1 (20 pts)



A single loop of wire is placed so it is completely immersed in a constant magnetic field B = 2200 mT $\hat{k}$. The wire is made of a revolutionary new material that is conducting, but will stretch to a large multiple of its original length. In this case the length L = 10 cm is held constant, but the width of the wire is controlled by the velocities $\mathbf{v}_1 = v_1\,\hat{\imath}$ and $\mathbf{v}_2 = v_2\,\hat{\imath}$ applied as shown. The width of the loop starts out at an initial length b. The wire has a resistance of 50 Ω, which can be assumed to be constant regardless of changes in the dimensions of the wire.

a) Determine an expression for the induced current in the wire if $v_1 = v_2 = v$. Calculate the magnitude and direction of the current in the wire loop at t = 5 s if v = 20 mm/s and b = 1 cm. (2 pts)
b) Determine an expression for the induced current in the wire if $v_1 < v_2$. Calculate the magnitude and direction of the current in the wire loop at t = 5 s if v_1 = 20 mm/s, v_2 = 25 mm/s and b = 1 cm. (4 pts)
c) Determine an expression for the induced current in the wire if $v_1 < v_2$ and v_2 is initially equal to v_1 and increasing linearly with time as βt. Calculate the magnitude and direction of the current in the wire loop at t = 5 s if v_1 = 20 mm/s, β = 1 mm/s^2 and b = 1 cm. (6 pts)
d) Determine an expression for the induced current in the wire if both ends start at rest and v_1 increases quadratically in time as γt^2 and v_2 increases linearly in time as βt. Plot the current in the wire loop vs. time using 0 s < t < 2 s if γ = 0.05 mm/s^3, β = 1 mm/s^2 and b = 10 cm. (8 pts)

a) $A = bL \Rightarrow \Phi_B = \int \vec{B}\cdot d\hat{A} = BA = BbL \Rightarrow \mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BbL) = \boxed{0}$

$\boxed{\therefore I = 0}$

b) $A = (b + (v_2 - v_1)t)L \Rightarrow \Phi_B = B[L(b + (v_2 - v_1)t)] = BLb + BL(v_2 - v_1)t$

$\Rightarrow \mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[BLb + BL(v_2 - v_1)t\right] = -BL(v_2 - v_1)$

$\Rightarrow I = \frac{\mathcal{E}}{R} = -\frac{BL(v_2 - v_1)}{R} = -\frac{(0.2\,T)(0.1\,m)[(0.025\,\frac{m}{s}) - (0.020\,\frac{m}{s})]}{50\,\Omega} = \boxed{-22\,\mu A}\boxed{CW}$

c) $v_2 = v_1 + \beta t$

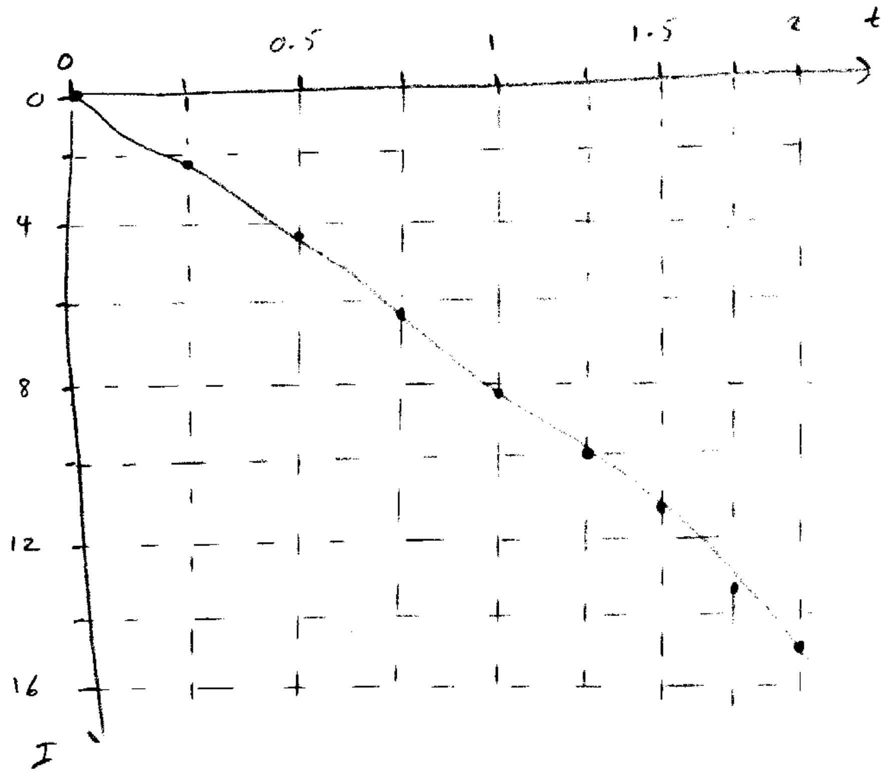
$A = [b + (v_2 - v_1)t]L \Rightarrow \Phi_B = BL(b + (v_2 - v_1)t) = BL(b + [(v_1 + \beta t) - v_1]t) = BLb + BL\beta t^2$

$\Rightarrow \mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BLb + BL\beta t^2) = -2BL\beta t \Rightarrow I = \frac{\mathcal{E}}{R} = -\frac{2BL\beta t}{R} = \boxed{-44\,\mu A\ CW}$

d) $v_1 = \gamma t^2,\ v_2 = \beta t \Rightarrow A = [b + (v_2 - v_1)t]L \Rightarrow \Phi_B = BL[b + (\gamma t^2 + \beta t)t] = BLb - BL\gamma t^3 + BL\beta t^2$

$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BLb - BL\gamma t^3 + BL\beta t^2) = -(-3BL\gamma t^2 + 2BL\beta t) = 3BL\gamma t^2 - 2BL\beta t$

$I = \frac{\mathcal{E}}{R} = \frac{3BL\gamma t^2 - 2BL\beta t}{R}$



$$I = \frac{3BL\gamma t^2}{R} - \frac{2BL\beta t}{R}$$

$$\Rightarrow \quad I = (6.6 \times 10^{-7})t^2 - (8.8 \times 10^{-6})t.$$

t	I (μA)
0	0
0.25	−2.16
0.5	−4.24
0.75	−6.23
1	−8.14
1.25	−9.97
1.5	−11.72
1.75	−13.38
2	−14.96

Problem 2 (20 pts)

An AC generator produces an emf described by $\varepsilon = (20\text{ V})\sin(300t)$. In addition to the generator the circuit contains a resistor having a resistance of 6.0 Ω and an unknown element all of which are in series. The RMS current in the circuit is 0.982 A. At t = 3.5 ms, the instantaneous current in the circuit is 1.098 A.

a) Find the maximum current and the phase constant for this circuit. (6 pts)
b) Identify the unknown element and determine the corresponding value of this element. (4 pts)
c) Find the impedance of the circuit and the average power dissipated by the circuit. (4 pts)
d) At what time after t = 0 s will ε and I first reach their respective maximum values? (4 pts)
e) Determine the value of a third element in series with the two already present that would give the circuit a resonance frequency of 377 rad/s, or indicate why this is not possible. (2 pts)

a) $I_{max} = \sqrt{2}\, I_{rms} = \sqrt{2}(0.982A) = \boxed{1.389\,A}$

$I(t) = I_{max}\sin(\omega t + \phi) \Rightarrow \phi = \sin^{-1}\left[\frac{I(t)}{I_{max}}\right] - \omega t = \sin^{-1}\left[\frac{1.098\,A}{1.389\,A}\right] - (300\tfrac{rad}{s})(3.5\times10^{-3}s)$

$\Rightarrow \boxed{\phi = -0.138\text{ rad}} = -7.92°$

b) at $t = 0$ $\phi < 0$ $\therefore$ $\boxed{\text{element is a capacitor}}$

$\tan\phi = \left(\frac{\cancel{X_L}^{0} - X_C}{R}\right) \Rightarrow X_C = -R\tan\phi = -(6\,\Omega)\tan(-0.138\text{ rad})$

$X_C = 0.833\,\Omega$

$\Rightarrow X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = 4\times10^{-3}\,F = 4\,mF$

c) $Z = \sqrt{R^2 + (\cancel{X_L} - X_C)^2} = \sqrt{(6\Omega)^2 + (-0.833\Omega)^2} = \boxed{6.06\,\Omega}$

$P_{avg} = \frac{1}{2}V_{max}I_{max}\cos\phi = \frac{1}{2}(20V)(1.389A)\cos(-0.138\text{ rad}) = \boxed{13.76\,W}$

d) $\mathcal{E}(t) = \mathcal{E}_{max} = \mathcal{E}_{max}\sin(\omega t) \Rightarrow 1 = \sin(\omega t) \Rightarrow \omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega}$

$\boxed{t = 5.24\,ms}$ → for voltage to reach max

$I(t) = I_{max} = I_{max}\sin(\omega t + \phi) = 1 = \sin(\omega t + \phi) \Rightarrow \omega t + \phi = \frac{\pi}{2}$

$\Rightarrow t = \frac{(\frac{\pi}{2} - \phi)}{\omega} = \frac{\frac{\pi}{2} - (-0.138)}{300} = \boxed{5.70\,ms}$ → for current to reach current max!

e) $\omega_0 = 377\frac{rad}{s}$

$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(377\frac{rad}{s})^2(4\,mF)}$ 5 $\Rightarrow \boxed{L = 1.76\,mH}$