PH 393 Physics II **Exam 1** **Fall 2011**

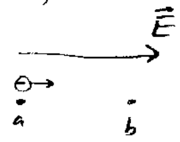
Name: Solutions **10/4/11**

Present neat and orderly answers for each question.
Clearly indicate your method of solution for each problem, including equations used.
Include appropriate units.
Show all work.

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / (N\ m^2)$
$k_e = 8.99 \times 10^9\ (N\ m^2) / C^2$

__Multiple Choice (2 pts each)__

1. A constant electric field is directed to the right. If a negative charge is placed in the region of the electric field and moved from point a to b in the direction of the electric field, which of the following describes the magnitude of the electric potential as the charge is moved from a to b.



   a. The electric potential is zero.
   b. The electric potential is constant.
   c. The electric potential increases.
   d. The electric potential decreases.

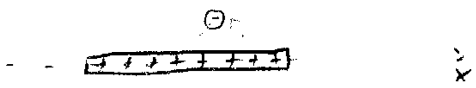
   Ans. C

2. Two charges are separated by a distance d, with the right charge being negative and the left charge being positive. If both charges are of the same magnitude (q), where could you place a positive test charge, along a single line that intersects both charges, with a charge magnitude that is q/1000 such that the force on the test charge would be zero (besides infinity)?

   a. To the right of both charges.
   b. In between the two charges.
   c. To the left of both charges.
   d. There is no location where the force would be zero.

   Ans. D

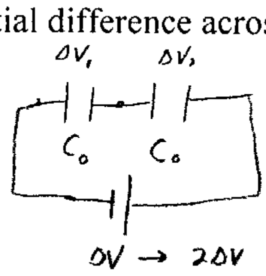
3. A positively charged rod is placed along the x-axis. A negatively charged particle moves in a circular arc around the rod/x-axis. The work done on the charged particle by the electric field of the rod is



   a. Positive.
   b. Negative.
   c. Zero.
   d. Not enough information to determine.

   Ans. C

4. What happens to the capacitance of each of the two identical capacitors, each with an initial capacitance C_0, connected in series when the potential difference across the circuit is doubled?



increasing potential difference doesn't change capacitance!

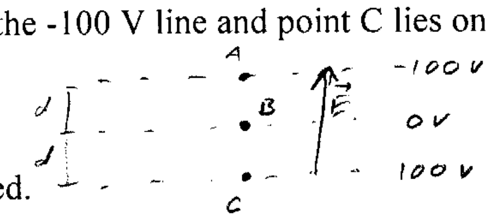
   a. $C = \frac{1}{4}C_0$;
   b. $C = \frac{1}{2}C_0$;
   c. $C = C_0$;
   d. $C = 2C_0$.

   Ans. C

5. A spherical metal shell can be used to store charge. If charge is added to the outer surface of the shell, how does the electric potential difference between the inner and outer surfaces change?
   a. The electric potential difference is zero.
   b. The electric potential difference is constant. ← non-zero !!
   c. The electric potential difference increases.
   d. The electric potential difference decreases.

   Ans. A

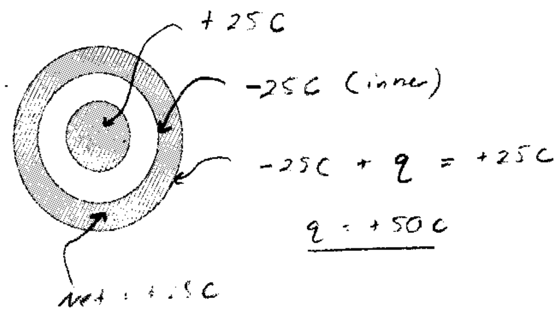
6. An electron is released from rest at point B, which lies along a 0 V equipotential line. A 100 V equipotential line lies to the right of the 0 V line and a -100 V equipotential line lies to the left of the 0 V line. All three equipotential lines are parallel and equidistant. Point A lies on the -100 V line and point C lies on the 100 V line. After it is released the electron



a. moves towards A with a constant speed.
b. moves towards A with an increasing speed.
c. moves towards C with a constant speed.
d. moves towards C with an increasing speed.

Ans. C

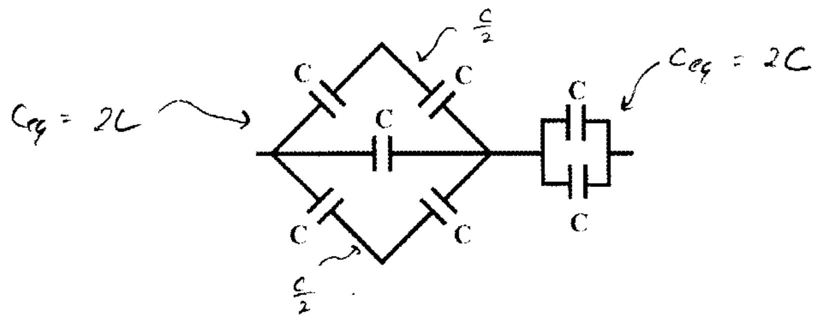
7. A solid conducting sphere has a net charge of *+25 C* and is surrounded by a conducting spherical shell having a net charge of *+25 C*. Both spheres are in electrostatic equilibrium. What is the charge on the spherical shell's outer surface?



a. – 25 C;
b. +25 C;
c. – 50 C;
d. + 50 C.

Ans. D

8. The circuit below contains two groups of capacitors. Group 1 is the group of 5 capacitors on the left and group 2 is the group of two capacitors on the right. Which of the two groups of capacitors can store more energy?



a. Group 1;
b. Group 2;
c. Both store the same amount of energy;
d. There is not enough information to decide.

Ans. C

9. Three charged metal spheres of different radii ($R_1 < R_2 < R_3$) are connected by a thin metal wire. The potential and electric field at the surface of each sphere are V and E. Which of the following correctly describes the relative sizes of the potentials and the electric fields?

a. $V_1=V_2=V_3$ and $E_1=E_2=E_3$
b. $V_1=V_2=V_3$ and $E_1>E_2>E_3$
c. $V_1>V_2>V_3$ and $E_1=E_2=E_3$
d. $V_1>V_2>V_3$ and $E_1>E_2>E_3$

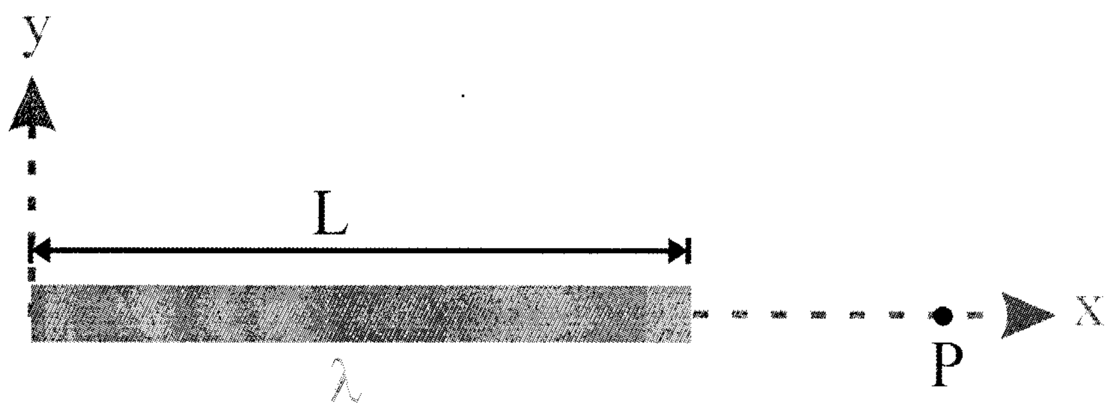
Ans. B

10. Where is the safest place to be during a lightning storm?

a. In a metal shed; → Assuming you are not touching the metal!
b. Under a tree;
c. On top of a tall building;
d. Lying flat on the ground.

Ans. A

Problem 1 (20 pts)



The rod of length L has a positive linear charge distribution $\lambda = \alpha x^3$, where α is a constant measured in C/m^4. Point P is an arbitrary point that lies along the x axis.

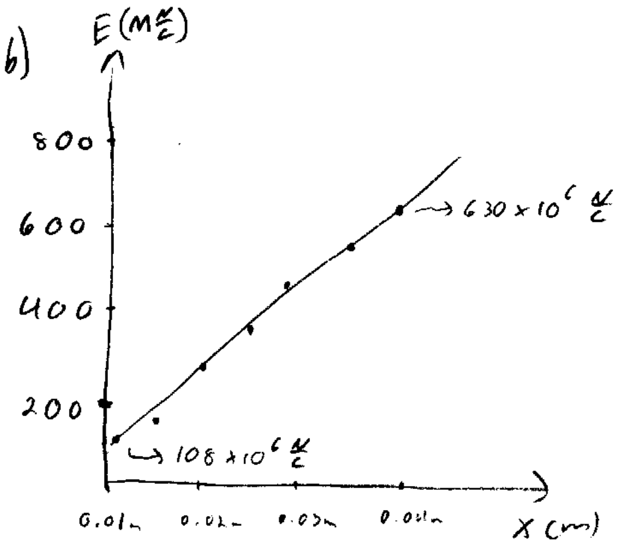
a) Determine an expression (including direction) for the electric field at point P as a function of horizontal position. (5 pts)
b) Plot the electric field strength vs. horizontal position. Use $\alpha = 200\ C/m^4$, L = 1 cm, and $1.1\text{ cm} \le x \le 4\text{ cm}$. Indicate the magnitude of the electric field at the limits. (3 pts)
c) Determine an expression for the electric potential at point P as a function of horizontal position. (5 pts)
d) Plot the electric potential vs. horizontal position. You can use $\alpha = 200\ C/m^4$, L = 1 cm, and $1.1\text{ cm} \le x \le 4\text{ cm}$. Indicate the magnitude of the electric field at the limits. (3 pts)
e) Determine the slope of the electric potential vs horizontal position curve for x = 2 cm and discuss what this value represents. (4 pts)

$dq = \lambda dx$

a) $E = \int \frac{k_e\, dq}{r^2} = k_e \int_{x-L}^{x} \frac{\lambda dx}{x^2} = k_e \int_{x-L}^{x} \frac{\alpha x^3}{x^2} dx = k_e \alpha \int_{x-L}^{x} x\, dx = \frac{k_e \alpha}{2} x^2 \Big|_{x-L}^{x} = \frac{k_e \alpha}{2}\left[x^2 - (x-L)^2\right]$

$= \left(\frac{k_e \alpha}{2}\right)\left[x^2 + 2xL - L^2 - x^2\right] = \frac{k_e \alpha}{2}\left[-L^2 + 2xL\right] \Rightarrow \boxed{\vec{E} = \frac{k_e \alpha}{2}\left[-L^2 + 2Lx\right]\hat{i}}$

b)



E (N/C)	x (m)
108×10^6	0.011
180×10^6	0.015
270×10^6	0.02
360×10^6	0.025
450×10^6	0.03
540×10^6	0.035
630×10^6	0.04

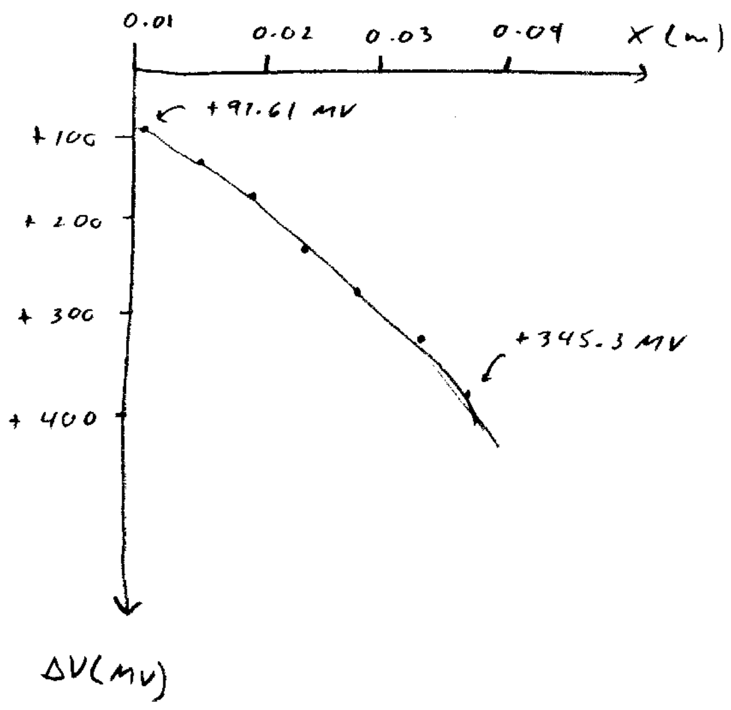
$E = \frac{k_e \alpha}{2}\left[L^2 + 2Lx\right] = 9\times10^{11}\left(-(0.01)^2 + 2(0.01)x\right)$

$(x-L)^3 = (x^2 - 2xL + L^2)(x-L)$
$= x^3 - x^2L - 2x^2L + 2xL^2 + xL^2 - L^3$
$= x^3 - 3x^2L + 3xL^2 - L^3$

c) $\Delta V = -\int \vec{E}\cdot d\vec{s} = -\int_{x-L}^{x} \frac{k_e \alpha}{2} x^2 dx = -\frac{k_e \alpha}{2}\left[\frac{x^3}{3}\right]_{x-L}^{x} = -\frac{k_e \alpha}{6}\left[x^3 - (x-L)^3\right] = -\frac{k_e \alpha}{6}\left[x^3 - x^3 + 3x^2L - 3xL^2 + L^3\right]$

$\Rightarrow \boxed{\Delta V = -\frac{k_e \alpha}{6}\left[3x^2L - 3xL + L^3\right]}$

d)



$$\Delta V = -\frac{k_e \alpha}{6}\left[3x^2L - 3xL + L^2\right]$$

$$= -(3\times10^{11})\left[3(0.01)x^2 - 3(0.01)x + (0.01)^3\right]$$

ΔV(MV)	x (m)
+97.61	0.011
+132.7	0.015
+176.1	0.02
+219.1	0.025
+261.6	0.03
+303.7	0.035
+345.3	0.04

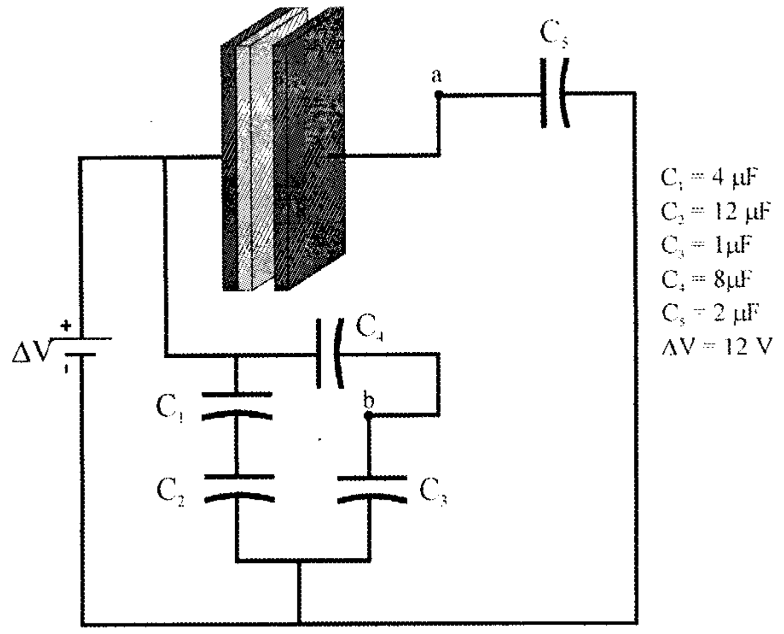
<u>Note</u>: sign error in my calculation.
Plot was drawn upside down!
Everything has been corrected

e) $\frac{d(\Delta V)}{dx} = E$ $\quad E = \frac{d}{dx}\left(\frac{k_e\alpha}{6}\left[x^3 - (x-L)^3\right]\right) = \frac{k_e\alpha}{6}\left[3x^2 - 3(x-L)^2\right] = \frac{k_e\alpha}{2}\left[x^2 - (x-L)^2\right]$

$$\boxed{E(0.02\,m) = 270\times10^6\ \frac{V}{m}}$$

The slope of the ΔV vs. x graph provides information about the Electric field.

Problem 2 (20 pts)



The capacitive circuit shown above consists of 5 standard capacitors and a pair of parallel plates. The plates are separated by a distance of 6.0 mm and the plates are 1.0 cm wide by 3.0 cm tall. A sheet of Mylar is placed between the two plates and in contact with the left plate. The Mylar sheet is 2.0 mm thick and has the same area as the plates. Mylar has a dielectric constant of 3.1 and a dielectric strength of $7.0\text{x}10^6$ V/m and air has a dielectric constant of 1.0 and a dielectric strength of $3.0\text{x}10^6$ V/m.

a) Find the equivalent capacitance for this circuit. (8 pts)
b) Find the charge on each side of the Mylar sheet. (5 pts)
c) Find the potential difference between points a and b. (5 pts)
d) Find the maximum potential difference that can be applied to the parallel plate capacitor and allow it to continue to work normally. (2 pts)

a)

$C_A = \frac{K_A \varepsilon_0 A}{d} = \frac{(1)\varepsilon_0 (0.01)(0.03)}{(0.004)} = 6.64\times10^{-13}$ F

$C_m = \frac{K_m \varepsilon_0 A}{d} = \frac{(3.1)(\varepsilon_0)(0.01)(0.03)}{(0.002)} = 4.12\times10^{-12}$ F

$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4\mu F} + \frac{1}{12\mu F} = \frac{4}{12\mu F} \Rightarrow C_{12} = 3\mu F$

$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{1\mu F} + \frac{1}{8\mu F} = \frac{9}{8\mu F} \Rightarrow C_{34} = 0.89\mu F$

$\frac{1}{C_{Am5}} = \frac{1}{C_A} + \frac{1}{C_m} + \frac{1}{C_5} = 1.75\times10^{12}\ \frac{1}{F}$

$\Rightarrow C_{Am5} = 5.72\times10^{-13} F = 5.72\times10^{-7}\mu F$

$C_{eq} = C_{12} + C_{34} + C_{Am5} = \boxed{3.89\ \mu F}$

b) $Q_{Am5} = C_{Am5}\Delta V = (5.72\times10^{-7}\mu F)(12 V)$

$Q_{Am5} = 6.86\times10^{-6}\ \mu C$

$Q_{Am5} = Q_5 = Q_A = \boxed{Q_m = 6.86\times10^{-6}\mu C}$

c) Assuming the negative terminal of the Battery is 0V.

$$V_a = \Delta V_5 = \frac{Q_5}{C_5} = \frac{6.86\times10^{-6}\,\mu C}{2\,\mu F} = 3.43\times10^{-6}\,V$$

$$Q_{34} = C_{34}\,\Delta V = (0.89\,\mu F)(12\,V) = 10.68\,\mu C \qquad Q_{34} = Q_4 = Q_3 = 10.68\,\mu C$$

$$V_b = \Delta V_3 = \frac{Q_{34}}{C_3} = \frac{10.68\,\mu C}{1\,\mu F} = 10.68\,V$$

$$\Delta V_{ab} = V_b - V_a = 10.68\,V - 3.43\times10^{-6}\,V$$

$$\Rightarrow \boxed{\Delta V_{ab} = 10.68\,V}$$

d) $\Delta V = Ed$ dielectric Strength $= E$!

Air:

$$\Delta V_A = E_a d_a = \left(3\times10^{6}\,\tfrac{V}{m}\right)(4\times10^{-3}\,m) = 12{,}000\,V$$

Mylar:

$$\Delta V_m = E_m d_m = \left(7\times10^{6}\,\tfrac{V}{m}\right)(2\times10^{-3}\,m) = 14{,}000\,V$$

$$\Delta V_T = \Delta V_A + \Delta V_m = 12{,}000\,V + 14{,}000\,V = 26{,}000\,V = \boxed{26\,KV}$$