Differentiation and integration of simple functions

<u>Differentiation:</u>

You differentiate a quantity to see how an infinitesimally small change in one quantity will affect the other. (For example, see how much y will change due to a small change in x)

Begin with an expression: $y = x^n$

Differentiate one quantity (y) with respect to the other (x): $\frac{dy}{dx} = \frac{d}{dx}\left(x^n\right) = n\left(x^{n-1}\right)$

<u>Note:</u> Differentiating a constant results in zero.

This is valid for any function that contains a power of a single quantity, or a sum (difference) of terms which each contain a power of a single quantity.

Examples:

$y = x^4$ $\qquad \frac{dy}{dx} = \frac{d}{dx}\left(x^4\right) = 4x^3$

$y = x^{12} + x^{-2} - x + 5$ $\qquad \frac{dy}{dx} = 12x^{11} - 2x^{-3} - 1 + 0$

Integration:

You integrate a quantity to determine how all of the infinitesimally small changes in a quantity (due to changes in another quantity) relate to the whole quantity you started with. (For example, you look at how all of the dy's relate to y, where all of the dy's are the result of an infinitesimally small change in x (dx). y = dy + dy + dy + … (y(x) = dy/dx + dy/dx + dy/dx + …)) *(area under the curve described by the expression for y)*

Begin with an expression: $y = x^n$

Integrate one quantity (y) with respect to the other (x):

$\int ydx = \int (x^n)dx = \frac{1}{n+1}(x^{n+1}) + c$, where c is a constant that must be identified. *(Indefinite integral)*

Integrate one quantity (y) with respect to the other (x) from some initial value of x (x_i) to some final value of x (x_f):

$$\int_{x_i}^{x_f} ydx = \int_{x_i}^{x_f} (x^n)dx = \frac{1}{n+1}(x^{n+1})\Big|_{x_i}^{x_f} = \frac{1}{n+1}\left[\left({x_f}^{n+1}\right) - \left({x_i}^{n+1}\right)\right]$$ *(Definite integral)*

Examples:

$y = x^4$ from $x_i = 2$ to $x_f = 6$

$$\int_2^6 ydx = \int_2^6 (x^4)dx = \frac{1}{5}(x^5)\Big|_2^6 = \frac{1}{5}\left[(6^5) - (2^5)\right] = \frac{1}{5}[7776 - 32] = 1548.8$$

$y = x^3 + x^{-2} - x + 5$ from $x_i = 2$ to $x_f = 6$

$$\int_2^6 ydx = \int_2^6 (x^3 + x^{-2} - x + 5)dx = \left(\frac{1}{4}x^4 - x^{-1} - \frac{1}{2}x^2 + 5x\right)\Big|_2^6$$

$$= \left[\left(\frac{1}{4}6^4 - 6^{-1} - \frac{1}{2}6^2 + 5(6)\right) - \left(\frac{1}{4}2^4 - 2^{-1} - \frac{1}{2}2^2 + 5(2)\right)\right]$$

$$= \left[(324 - 0.1667 - 18 + 30) - (4 - 0.5 - 2 + 10)\right]$$

$$= \left[(335.83) - (11.5)\right] = 324.3$$