Math Scramble

Emporia State University Math Day

October 23, 2019

Math Scramble Rules

- Equations should be solved over the real number system (*not* the complex numbers), unless specified otherwise.

- All answers must be exact (no decimal approximations), unless specified otherwise.

- Units must be included whenever possible.

- Calculators are allowed.

- Diagrams are not drawn to scale.

What is the area of a right triangle whose hypotenuse is 39 and ratio of its legs is $5 : 12$?

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Answer: 270

Including the 8.97% sales tax, a shopper at a hardware store paid a total of $\$30.25$ for a shovel. What was the sticker price of the shovel, rounded to the nearest penny?

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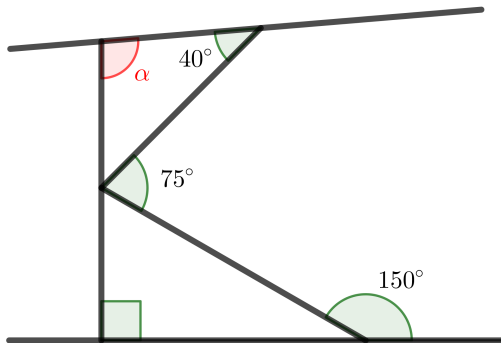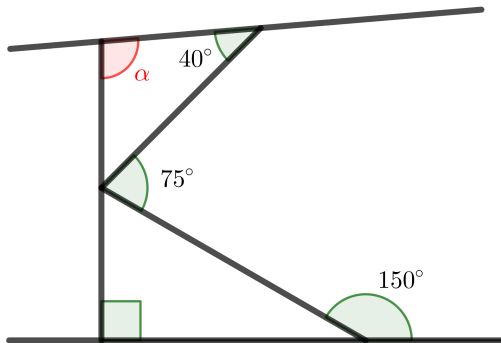
Answer: $\$27.76$

Find the angle α.

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Answer: $95°$

Express the repeating decimal

$$0.3\overline{215} = 0.3215215\ldots$$

as a fraction that is reduced to lowest terms.

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Answer: $\dfrac{1606}{4995}$

The average of six numbers is 36. If one number is removed, the average becomes 30. What is the number that was removed?

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Answer: 66
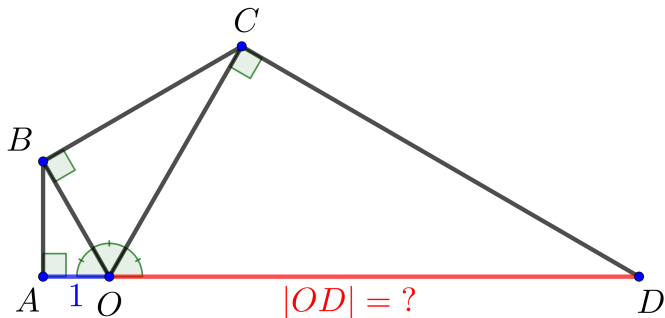
The point O lies on the line AD, $|OA| = 1$, and the right

triangles AOB, BOC, and COD are similar. Find $|OD|$.

$|OD| = ?$

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triangles AOB, BOC, and COD are similar. Find $|OD|$.

Answer: 8

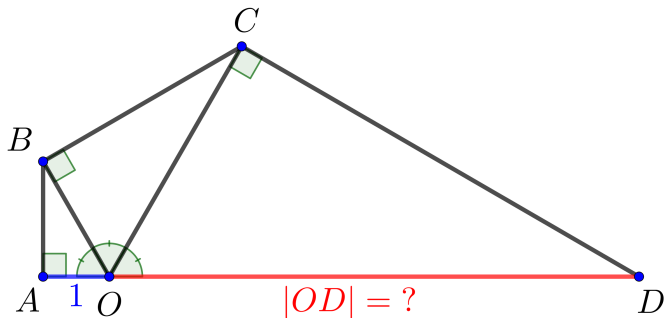For each positive integer n, let $s(n)$ denote the sum of the digits in the decimal representation of n. For example, $s(24) = 2 + 4 = 6$.

Let $s^2(n) = s(s(n))$, $s^3(n) = s(s(s(n)))$, and so forth.

What is the value of $s^{2019}(2019)$?

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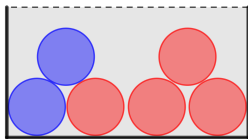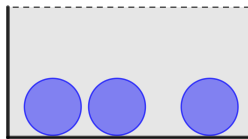
What is the value of $s^{2019}(2019)$?

Answer: 3

Bag 1 contains 2 blue marbles and 4 red marbles. Bag 2 contains 3 blue marbles. One marble is randomly chosen from Bag 1 and placed in Bag 2. Then one marble is randomly chosen from Bag 2. What is the probability that the marble chosen from Bag 2 is red? State your answer as a reduced fraction.

Bag 1   Bag 2

Answer: $\frac{1}{6}$

Solve for x:

$$\log_2(x) - \log_2(3) = 4$$

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Answer: 48

What is the value of

$$\frac{\log_b(a)}{\log_b\left(\frac{1}{a}\right)},$$

where $a > 0$ and $b > 0$ with $b \neq 1$?

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$$\frac{\log_b(a)}{\log_b\left(\frac{1}{a}\right)},$$

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Answer: -1

A total of $1,000$ is invested. Part of the money is invested in a mutual fund earning 3.5% per year, and the rest of the money is invested in a savings account earning 1% per year. (Both rate are stated as simple interest.) After one year, the total investment brings a return of $20. How much was invested in the mutual fund?

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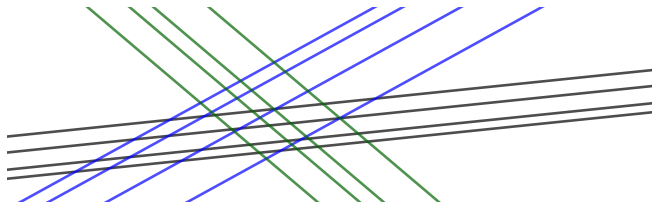
Answer: $\$400$

Consider 12 lines in a plane, consisting of 3 sets of 4 parallel

lines. If you randomly choose three of the lines, what is the

probability that at least two of the chosen lines intersect?

Express your answer as a reduced fraction.

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lines. If you randomly choose three of the lines, what is the

probability that at least two of the chosen lines intersect?

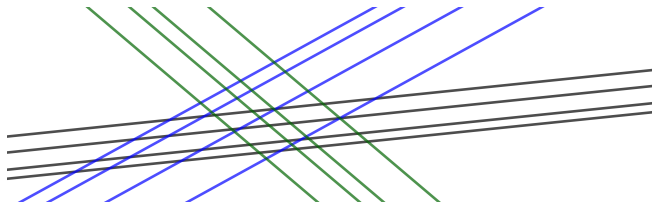Express your answer as a reduced fraction.

Answer: $\dfrac{52}{55}$

Find the smallest $x > 0$ satisfying the
following equation, where x is in radians:

$$5\cos(x) + 2\sin^2(x) = 4$$

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$$5\cos(x) + 2\sin^2(x) = 4$$

Answer: $\dfrac{\pi}{3}$

The annual revenue, in dollars, for selling x units of a product is given by

$$R(x) = 1000 + 100x - 0.025x^2.$$

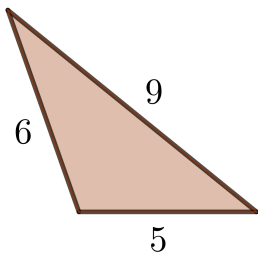Find the maximum possible annual revenue.

The annual revenue, in dollars, for selling x units of a product is given by

$$R(x) = 1000 + 100x - 0.025x^2.$$

Find the maximum possible annual revenue.

Answer: $101,000

Find the area of the triangle.

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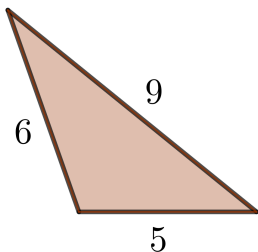
Answer: $10\sqrt{2}$

You have some dimes and quarters worth a total of 7.15. You have three times as many dimes as quarters. How many coins do you have?

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Answer: 52

If the digits in the number 4125 are randomly shuffled, what is the probability that the resulting number is divisible by 6? Express your answer as a reduced fraction.

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Answer: $\dfrac{1}{2}$

A sphere has a volume of V, and each of its great circles have a circumference of C. Find the number $\dfrac{C^3}{V}$.

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Answer: $6\pi^2$

Sally runs one mile at 6 miles per hour and then the next mile at 8 miles per hour. What is her average speed for the 2-mile run? State your answer with units of miles per hour.

Sally runs one mile at 6 miles per hour and then the next mile at 8 miles per hour. What is her average speed for the 2-mile run? State your answer with units of miles per hour.

Answer: $\dfrac{48}{7}$ m.p.h. (i.e., $6\frac{6}{7}$ m.p.h.)

Using the standard 26 letter English alphabet, how many strings of 4 letters contain exactly one vowel? Assume that the sets of vowels is $\{a, e, i, o, u\}$.

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Answer: $185{,}220$

Simplify, assuming that $x > 0$:

$$\frac{x\sqrt{4 + \left(x - \frac{1}{x}\right)^2}}{x^2 + 1}$$

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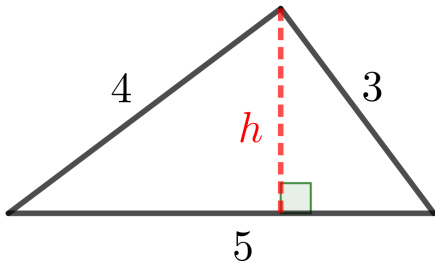$$\frac{x\sqrt{4 + \left(x - \frac{1}{x}\right)^2}}{x^2 + 1}$$
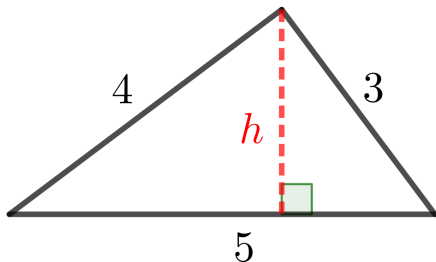
Answer: 1

Find the height h of the triangle.

Find the height h of the triangle.

Answer: $\dfrac{12}{5}$ (i.e., $2\frac{2}{5}$ or 2.4)

If the price of an item is increased by 20% three times in a row, what is the percentage increase of the final price to the original price?

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Answer: 72.8%

A group of students take a test, and their average score is 74. If one more student had taken the test and scored 100, then the average score would have been 74.5. How many students took the test?

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Answer: 51

Tiebreakers

Solve $|x - 4| = x$ for x.

Solve $|x - 4| = x$ for x.

Answer: 2

What is the smallest positive integer that has five distinct prime factors?

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Answer: 2310

Express the solution set of the inequality

$$\frac{1}{x} + 2x \geq 3$$

in interval notation.

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$$\frac{1}{x} + 2x \geq 3$$

in interval notation.

Answer: $(0, \frac{1}{2}] \cup [1, \infty)$